

Certain Level Operators over Temporal Intuitionistic Fuzzy Sets

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ABSTRACT: Intuitionistic Fuzzy Sets are substantial extensions of fuzzy sets which play a key factor in describing and providing ease of solving higher complexities in engineering and science. However, certain ambiguous situations cannot be addressed by using fuzzy sets and intuitionistic fuzzy sets. The extension of fuzzy set, namely, Temporal Intuitionistic Fuzzy Sets paved the way for implementing methods and techniques for unanswered problems. In this study, we proposed certain level operators $N_\alpha(A)$, $N^\alpha(A)$ and $N_{\alpha,\beta}(A)$ for temporal intuitionistic fuzzy sets and investigated some of their properties. We gave numerical examples for TIFS and also study the relations between temporal intuitionistic fuzzy sets of certain level.

KEYWORDS: Fuzzy Sets, Intuitionistic Fuzzy Sets, Temporal Intuitionistic Fuzzy Sets, Necessity operator and Possibility operator.

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1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [1] in as an extension of crisp sets by expanding the truth value set to the real unit interval $[0, 1]$. Let X be a non empty set. The function $\mu_A: X \rightarrow [0,1]$ is called a fuzzy set over X . For $x \in X$, $\mu(x)$ is the membership degree of x and the non-membership degree is $1 - \mu(x)$. Intuitionistic Fuzzy Sets (IFS) have been introduced by Atanassov [2] which is an extension of fuzzy sets. If X is a universal then a intuitionistic fuzzy set A , the membership and non-membership degree for each $x \in X$ respectively, $\mu_A(x)$ and $\nu_A(x)$ Such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1$. The class of intuitionistic fuzzy sets on X is denoted by IFS. While the sum of membership degree and non-membership degree is 1 on FS, this sum is less than 1 on IFS.

Another important extension of fuzzy sets is the temporal intuitionistic fuzzy set obtained by adding the time parameter to the intuitionistic fuzzy set [3]. It is obvious that the idea of changing the degree of membership and degree of non-membership over time and location has created a rich field of study in spatio – temporal research fields such as weather, economy, image processing and video processing. On the other hand, effective results have been obtained in many different ways on the mentioned subjects. The fact that the concept of the temporal intuitionistic fuzzy sets is to be

obtained by taking into account the time parameter is still not defined in the literature constitutes a great deficiency. After this extension author shows that all operations and operators on the intuitionistic fuzzy sets can be defined for the temporal intuitionistic fuzzy sets. In 2009, Parvathi and Geetha [4] defined some level operators, max-min implication operators and $P_{\alpha,\beta}$, $Q_{\alpha,\beta}$ operators on temporal intuitionistic fuzzy sets.

In 2016, Rajesh and Srinivasan [5,6,7] studied the properties of temporal intuitionistic fuzzy sets of second type and also introduced various distance measure. In 2019, Khalaf e tal., [8], introduced Similarity measures between temporal complex intuitionistic fuzzy sets and application in pattern recognition and medical diagnosis, Kutlu and Bilgin [9] introduced the Temporal intuitionistic fuzzy topology in Šostak's sense in 2015. In 2014, Yilmaz and Cuvalcioglu [10] defined the level operators for temporal intuitionistic fuzzy sets.

In this paper we have introduced the temporal intuitionistic fuzzy sets of certain level and some operations and relations over temporal intuitionistic fuzzy sets of certain level.

PRELIMINARIES

In this section, we give some basic definitions.

Definition 2.1[10] Let X be a non empty set. A Fuzzy Set (IFS) A in X is defined as an object of

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the following form.

$$A = \{ \langle x, \mu_A(x) \rangle \mid x \in X \}$$

Where the functions $\mu_A: X \rightarrow [0,1]$ define the degree of membership of the element $x \in X$ and for every $x \in X$.

$$0 \leq \mu_A(x) \leq 1$$

Definition 2.2[11] An Intuitionistic Fuzzy Set (IFS) A in X is defined as an object of the following form.

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \}$$

Where the functions $\mu_A: X \rightarrow [0,1]$ and $\nu_A: X \rightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$, respectively, and for every $x \in X$.

$$0 \leq \mu_A(x) + \nu_A(x) \leq 1$$

The ordinary fuzzy set may be written as $A = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle \mid x \in X \}$

Definition 2.3[12] (Intuitionistic Fuzzy Sets of a Certain Level) A set of (α, β) - level, generated by an IFS A , where $\alpha, \beta \in [0, 1]$ are fixed number such that $\alpha + \beta \leq 1$ is defined as

$$N_{\alpha, \beta}(A) = \{ \langle x, \mu_A(x), \nu_A(x) \rangle \mid x \in X \text{ } \mu_A(x) \geq \alpha \text{ } \nu_A(x) \leq \beta \}$$

Definition 2.4 [13] Let X be a non empty set. For every IFS A , the necessity operator and possibility operator are defined in the following form

Necessity operator

$$\Box A = \{ \langle x, \mu_A(x), 1 - \mu_A(x) \rangle \mid x \in X \}$$

Possibility operator

$$\Diamond A = \{ \langle x, 1 - \nu_A(x), \nu_A(x) \rangle \mid x \in X \}$$

Definition 2.5[8] Temporal Intuitionistic Fuzzy Sets (TIFS) is defined as the following object of the form

$$A = \{ \langle x, \mu_A(x, t), \nu_A(x, t) \rangle \mid \langle x, t \rangle \in X \times T \}$$

Where

- i) $A \subset X$ is a fixed set.
- ii) $\mu_A(x, t) + \nu_A(x, t) \leq 1$ For every $\langle x, t \rangle \in X \times T$.
- iii) $\mu_A(x, t)$ and $\nu_A(x, t)$ are the degree of membership and the degree of non-membership of the element $x \in X$, respectively, and for every $x \in E$ at the time - moment $t \in T$

1. Temporal Intuitionistic Fuzzy Sets of Certain Level

In this section we introduce the certain level operators over TIFS

Definition 3.1 A set of (α, β) - level, generated by an TIFS A , where $\alpha, \beta \in [0, 1]$ are fixed number such that $\alpha + \beta \leq 1$ is defined as

$$N_{\alpha, \beta}(A) = \{ \langle x, \mu_A(x, t), \nu_A(x, t) \rangle \mid x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \text{ } \nu_A(x, t) \leq \beta \}$$

Definition 3.2 The set $N_{\alpha}(A) = \{ \langle x, \mu_A(x, t), \nu_A(x, t) \rangle \mid x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \}$

Is called a set of level of membership α generated by A .

Definition 3.3 The set $N^{\alpha}(A) = \{ \langle x, \mu_A(x, t), \nu_A(x, t) \rangle \mid x \in X \text{ } t \in T \text{ } \nu_A(x, t) \leq \alpha \}$

Is called a set of level of non-membership α generated by A .

Example 3.4 Let $X = \{a, b, c, d, e\}$. Let the TIFSCL A has the form.

$$A = \{ \langle a, 0.5, 0.3 \rangle, \langle b, 0.1, 0.7 \rangle, \langle c, 1.0, 0.0 \rangle, \langle d, 0.0, 0.0 \rangle, \langle e, 0.0, 1.0 \rangle \}$$

Then

$$1. N_{\alpha, \beta}(A) = \{ \langle x, \mu_A(x, t), \nu_A(x, t) \rangle \mid x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \text{ } \nu_A(x, t) \leq \beta \}$$

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- $$N_{\alpha,\beta}(A) = \{ \langle a, 0.5, 0.3 \rangle, \langle c, 1.0, 0.0 \rangle \}$$
- $$2. N^{\alpha}(A) = \{ \langle x, \mu_A(x, t), v_A(x, t) \rangle \mid x \in X \text{ } t \in T \text{ } v_A(x, t) \leq \alpha \}$$
- $$N^{0.4}(A) = \{ \langle a, 0.5, 0.3 \rangle, \langle c, 0.5, 0.5 \rangle, \langle d, 0.0, 0.0 \rangle \}$$
- $$3. N_{\alpha}(A) = \{ \langle x, \mu_A(x, t), v_A(x, t) \rangle \mid x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \}$$
- $$N_{0.3}(A) = \{ \langle a, 0.5, 0.3 \rangle, \langle c, 1.0, 0.0 \rangle \}$$

4. Some Operations And Relations Over Temporal Intuitionistic Fuzzy Sets of Certain Level

For the two TIFSCL A and B we define the following basic relations and operations.

- $$1. N_{\alpha,\beta}(A) \cap N_{\alpha,\beta}(B) = \{ \langle x, \min(\mu_A(x, t), \mu_B(x, t)), \max(v_A(x, t), v_B(x, t)) \rangle \mid x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \text{ } v_A(x, t) \leq \beta \}$$
- $$2. N_{\alpha,\beta}(A) \cup N_{\alpha,\beta}(B) = \{ \langle x, \max(\mu_A(x, t), \mu_B(x, t)), \min(v_A(x, t), v_B(x, t)) \rangle \mid x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \text{ } v_A(x, t) \leq \beta \}$$
- $$3. N_{\alpha,\beta}(\bar{A}) = \{ \langle x, v_A(x, t), \mu_A(x, t) \rangle \mid x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \text{ } v_A(x, t) \leq \beta \}$$
- $$4. N_{\alpha,\beta}(A) + N_{\alpha,\beta}(B) = \{ \langle x, \mu_A(x, t) + \mu_B(x, t) - \mu_A(x, t) \cdot \mu_B(x, t), v_A(x, t) \cdot v_B(x, t) \rangle \mid x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \text{ } v_A(x, t) \leq \beta \}$$
- $$5. N_{\alpha,\beta}(A) \cdot N_{\alpha,\beta}(B) = \{ \langle x, \mu_A(x, t) \cdot \mu_B(x, t), v_A(x, t) + v_B(x, t) - v_A(x, t) \cdot v_B(x, t) \rangle \mid x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \text{ } v_A(x, t) \leq \beta \}$$
- $$6. N_{\alpha,\beta}(A) @ N_{\alpha,\beta}(B) = \left\{ \left\langle x, \frac{\mu_A(x, t) \cdot \mu_B(x, t)}{2}, \frac{v_A(x, t) + v_B(x, t)}{2} \right\rangle \mid x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \text{ } v_A(x, t) \leq \beta \right\}$$
- $$7. \square N_{\alpha,\beta}(A) = \{ \langle x, \mu_A(x, t), 1 - \mu_A(x, t) \rangle \mid x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \}$$
- $$8. \diamond N_{\alpha,\beta}(A) = \{ \langle x, 1 - v_A(x, t), v_A(x, t) \rangle \mid x \in X \text{ } t \in T \text{ } v_A(x, t) \leq \alpha \}$$

Example 4.1: Let $X = \{a, b, c, d, e\}$. Let the TIFSCL A has the form.

$$A = \{ \langle a, 0.5, 0.3 \rangle, \langle b, 0.1, 0.7 \rangle, \langle c, 1.0, 0.0 \rangle, \langle d, 0.0, 0.0 \rangle, \langle e, 0.0, 1.0 \rangle \}$$

$$B = \{ \langle a, 0.7, 0.1 \rangle, \langle b, 0.3, 0.2 \rangle, \langle c, 0.5, 0.5 \rangle, \langle d, 0.2, 0.2 \rangle, \langle e, 1.0, 0.0 \rangle \}$$

Then

$$(i) N_{\alpha,\beta}(A) \cap N_{\alpha,\beta}(B) = \{ \langle x, \min(\mu_A(x, t), \mu_B(x, t)), \max(v_A(x, t), v_B(x, t)) \rangle \mid x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \text{ } v_A(x, t) \leq \beta \}$$

$$= \{ \langle a, \min(0.5, 0.7), \max(0.3, 0.1) \rangle, \langle b, \min(0.1, 0.3), \max(0.7, 0.2) \rangle, \langle c, \min(1.0, 0.5), \max(0.0, 0.5) \rangle, \langle d, \min(0.0, 0.2), \max(0.0, 0.2) \rangle, \langle e, \min(0.0, 1.0), \max(1.0, 0.0) \rangle \}$$

$$= \{ \langle a, 0.5, 0.3 \rangle, \langle b, 0.1, 0.7 \rangle, \langle c, 0.5, 0.5 \rangle, \langle d, 0.0, 0.2 \rangle, \langle e, 0.0, 1.0 \rangle \}$$

If $\alpha = 0.3$ and $\beta = 0.4$ then the conditions $\mu_A(x, t) \geq 0.3 \text{ } v_A(x, t) \leq 0.4$

$$N_{0.3,0.4}(A) \cap N_{0.3,0.4}(B) = \{ \langle a, 0.5, 0.3 \rangle \}$$

$$(ii) N_{\alpha,\beta}(A) \cup N_{\alpha,\beta}(B) = \{ \langle x, \max(\mu_A(x, t), \mu_B(x, t)), \min(v_A(x, t), v_B(x, t)) \rangle \mid x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \text{ } v_A(x, t) \leq \beta \}$$

$$= \{ \langle a, 0.7, 0.1 \rangle, \langle b, 0.3, 0.2 \rangle, \langle c, 1.0, 0.0 \rangle, \langle d, 0.2, 0.0 \rangle, \langle e, 1.0, 0.0 \rangle \}$$

If $\alpha = 0.3$ and $\beta = 0.4$ then the conditions $\mu_A(x, t) \geq 0.3 \text{ } v_A(x, t) \leq 0.4$

$$N_{0.3,0.4}(A) \cup N_{0.3,0.4}(B) = \{ \langle a, 0.7, 0.1 \rangle, \langle b, 0.3, 0.2 \rangle, \langle c, 1.0, 0.0 \rangle, \langle e, 1.0, 0.0 \rangle \}$$

$$(iii) N_{\alpha,\beta}(\bar{A}) = \{ \langle x, v_A(x, t), \mu_A(x, t) \rangle \mid x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \text{ } v_A(x, t) \leq \beta \}$$

$$= \{ \langle a, 0.3, 0.5 \rangle, \langle b, 0.7, 0.1 \rangle, \langle c, 0.0, 1.0 \rangle, \langle d, 0.0, 0.0 \rangle, \langle e, 1.0, 0.0 \rangle \}$$

If $\alpha = 0.3$ and $\beta = 0.4$ then the conditions $\mu_A(x, t) \geq 0.3 \text{ } v_A(x, t) \leq 0.4$

$$N_{0.3,0.4}(\bar{A}) = \{ \langle b, 0.7, 0.1 \rangle, \langle e, 1.0, 0.0 \rangle \}$$

$$(iv) N_{\alpha,\beta}(A) + N_{\alpha,\beta}(B) = \{ \langle x, \mu_A(x, t) + \mu_B(x, t) - \mu_A(x, t) \cdot \mu_B(x, t), v_A(x, t) \cdot v_B(x, t) \rangle \mid x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \text{ } v_A(x, t) \leq \beta \}$$

$$= \{ \langle a, 0.85, 0.03 \rangle, \langle b, 0.37, 0.14 \rangle, \langle c, 1.0, 0.0 \rangle, \langle d, 0.2, 0.0 \rangle, \langle e, 1.0, 0.0 \rangle \}$$

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If $\alpha = 0.3$ and $\beta = 0.4$ then the conditions $\mu_A(x, t) \geq 0.3$ & $v_A(x, t) \leq 0.4$

$$N_{0.3,0.4}(A) + N_{0.3,0.4}(B) = \{ \langle a, 0.85, 0.03 \rangle, \langle b, 0.37, 0.14 \rangle, \langle c, 1.0, 0.0 \rangle, \langle e, 1.0, 0.0 \rangle \}$$

$$\begin{aligned} (v) \quad N_{\alpha,\beta}(A) \cdot N_{\alpha,\beta}(B) &= \{ \langle x, \mu_A(x, t) \cdot \mu_B(x, t), v_A(x, t) + v_B(x, t) - v_A(x, t) \cdot v_B(x, t) \rangle \\ &\quad | x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \text{ } v_A(x, t) \leq \beta \} \\ &= \{ \langle a, 0.35, 0.37 \rangle, \langle b, 0.03, 0.76 \rangle, \langle c, 0.5, 0.5 \rangle, \langle d, 0.0, 0.2 \rangle, \langle e, 0.0, 1.0 \rangle \} \end{aligned}$$

If $\alpha = 0.3$ and $\beta = 0.4$ then the conditions $\mu_A(x, t) \geq 0.3$ & $v_A(x, t) \leq 0.4$

$$N_{0.3,0.4}(A) \cdot N_{0.3,0.4}(B) = \{ \langle a, 0.35, 0.37 \rangle \}$$

$$\begin{aligned} (vi) \quad N_{\alpha,\beta}(A) @ N_{\alpha,\beta}(B) &= \left\{ \left\langle x, \frac{\mu_A(x, t) \cdot \mu_B(x, t)}{2}, \frac{v_A(x, t) + v_B(x, t)}{2} \right\rangle \right. \\ &\quad \left. | x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \text{ } v_A(x, t) \leq \beta \right\} \\ &= \{ \langle a, 0.6, 0.2 \rangle, \langle b, 0.2, 0.45 \rangle, \langle c, 0.75, 0.25 \rangle, \langle d, 0.1, 0.1 \rangle, \langle e, 0.5, 0.5 \rangle \} \end{aligned}$$

If $\alpha = 0.3$ and $\beta = 0.4$ then the conditions $\mu_A(x, t) \geq 0.3$ & $v_A(x, t) \leq 0.4$

$$N_{0.3,0.4}(A) @ N_{0.3,0.4}(B) = \{ \langle a, 0.6, 0.2 \rangle, \langle c, 0.75, 0.25 \rangle \}$$

$$\begin{aligned} (vii) \quad \square N_{\alpha,\beta}(A) &= \{ \langle x, \mu_A(x, t), 1 - \mu_A(x, t) \rangle | x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \} \\ &= \{ \langle a, 0.5, 0.5 \rangle, \langle b, 0.1, 0.99 \rangle, \langle c, 1.0, 0.0 \rangle, \langle d, 0.0, 1.0 \rangle, \langle e, 0.0, 1.0 \rangle \} \end{aligned}$$

If $\alpha = 0.3$ and $\beta = 0.4$ then the conditions $\mu_A(x, t) \geq 0.3$ & $v_A(x, t) \leq 0.4$

$$\square N_{\alpha,\beta}(A) = \{ \langle c, 1.0, 0.0 \rangle \}$$

$$\begin{aligned} (viii) \quad \diamond N_{\alpha,\beta}(A) &= \{ \langle x, 1 - v_A(x), v_A(x) \rangle | x \in X \text{ } t \in T \text{ } v_A(x, t) \leq \alpha \} \\ &= \{ \langle a, 0.7, 0.3 \rangle, \langle b, 0.3, 0.7 \rangle, \langle c, 1.0, 0.0 \rangle, \langle d, 1.0, 0.0 \rangle, \langle e, 0.0, 1.0 \rangle \} \end{aligned}$$

If $\alpha = 0.3$ and $\beta = 0.4$ then the conditions $\mu_A(x, t) \geq 0.3$ & $v_A(x, t) \leq 0.4$

$$\diamond N_{\alpha,\beta}(A) = \{ \langle a, 0.7, 0.3 \rangle, \langle c, 1.0, 0.0 \rangle, \langle d, 1.0, 0.0 \rangle \}$$

Therefore A, B is the temporal Intuitionistic Fuzzy Sets of a Certain Level (TIFSCL).

Theorem: 4.2 Let X be a non empty set. For every TIFS of a Certain level A in X.

$$(i) \quad N_{\alpha,\beta}(A) = \square N_{\alpha,\beta}(A) \quad (ii) \quad N_{\alpha,\beta}(A) = \diamond N_{\alpha,\beta}(A) \quad (iii) \quad \square N_{\alpha,\beta}(A) = \diamond N_{\alpha,\beta}(A)$$

Proof: Proof (i)

By the definition

$$N_{\alpha,\beta}(A) = \{ \langle x, \mu_A(x, t), v_A(x, t) \rangle | x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \text{ } v_A(x, t) \leq \beta \}$$

Apply the necessity operator we've

$$\square N_{\alpha,\beta}(A) = \{ \langle x, \mu_A(x, t), 1 - \mu_A(x, t) \rangle | x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \}$$

Since, $\mu_A(x) + v_A(x) \leq 1$, this implies that $\mu_A(x) \leq 1 - v_A(x)$ and $v_A(x) \leq 1 - \mu_A(x)$

$$\begin{aligned} \square N_{\alpha,\beta}(A) &= \{ \langle x, \mu_A(x, t), v_A(x, t) \rangle | x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha, v_A(x, t) \leq \beta \} \\ &= N_{\alpha,\beta}(A) \end{aligned}$$

Hence, $N_{\alpha,\beta}(A) = \square N_{\alpha,\beta}(A)$.

Proof (ii) By the definition

$$N_{\alpha,\beta}(A) = \{ \langle x, \mu_A(x, t), v_A(x, t) \rangle | x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \text{ } v_A(x, t) \leq \beta \}$$

Apply the possibility operator in the above equation we've

$$\diamond N_{\alpha,\beta}(A) = \{ \langle x, 1 - v_A(x), v_A(x) \rangle | x \in X \text{ } t \in T \text{ } v_A(x, t) \leq \alpha \}$$

Since $\mu_A(x) + v_A(x) \leq 1$ this implies that $\mu_A(x) \leq 1 - v_A(x)$ and $v_A(x) \leq 1 - \mu_A(x)$

$$\begin{aligned} \diamond N_{\alpha,\beta}(A) &= \{ \langle x, \mu_A(x, t), v_A(x, t) \rangle | x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha, v_A(x, t) \leq \beta \} \\ \diamond N_{\alpha,\beta}(A) &= N_{\alpha,\beta}(A) \end{aligned}$$

Hence $N_{\alpha,\beta}(A) = \diamond N_{\alpha,\beta}(A)$

Proof (iii): Comparing the above two proofs (i) and (ii) we've, $\square N_{\alpha,\beta}(A) = \diamond N_{\alpha,\beta}(A)$

Theorem: 4.3 Let X be a non empty set. For every TIFS of a Certain level A in X.

$$N_{\alpha,\beta}(A) = \square N_{\alpha,\beta}(A)$$

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Proof: By the definition

$$N_{\alpha,\beta}(A) = \{ \langle x, \mu_A(x, t), v_A(x, t) \rangle \mid x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \text{ } v_A(x, t) \leq \beta \}$$

Apply the necessity operator we've

$$\Box N_{\alpha,\beta}(A) = \{ \langle x, \mu_A(x, t), 1 - \mu_A(x, t) \rangle \mid x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha \}$$

Since $\mu_A(x) + v_A(x) \leq 1$ this implies that $\mu_A(x) \leq 1 - v_A(x)$ and $v_A(x) \leq 1 - \mu_A(x)$

$$\Box N_{\alpha,\beta}(A) = \{ \langle x, \mu_A(x, t), v_A(x, t) \rangle \mid x \in X \text{ } t \in T \text{ } \mu_A(x, t) \geq \alpha, v_A(x, t) \leq \beta \}$$

$$= N_{\alpha,\beta}(A)$$

Hence $N_{\alpha,\beta}(A) = \Box N_{\alpha,\beta}(A)$.

Theorem: 4.4 Let X be a non empty set. For every TIFS of a Certain level A in X .

- i) $\Box \Box N_{\alpha,\beta}(A) = \Box N_{\alpha,\beta}(A)$
- ii) $\Diamond \Diamond N_{\alpha,\beta}(A) = \Diamond N_{\alpha,\beta}(A)$
- iii) $\Diamond \Box N_{\alpha,\beta}(A) = N_{\alpha,\beta}(A)$
- iv) $\Box \Diamond N_{\alpha,\beta}(A) = N_{\alpha,\beta}(A)$

Proof: The proof is similar.

5. CONCLUSION

In this study, we proposed certain level operators $N_\alpha(A)$, $N^\alpha(A)$ and $N_{\alpha,\beta}(A)$ for temporal intuitionistic fuzzy sets and investigated some of their properties and also given a numerical examples for over temporal intuitionistic fuzzy sets. Finally we established their relations between temporal intuitionistic fuzzy sets of certain level. It is still open to define some more operations and operators over temporal intuitionistic fuzzy sets. Further research work is in progress to solve the higher complexities in engineering and science. However, the way for implementing methods and techniques for unanswered problems using temporal intuitionistic fuzzy sets.

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